2024 AMC 10B Problems

Problem 1

In a long line of people, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

$\textbf{(A) } 2021 \qquad \textbf{(B) } 2022 \qquad \textbf{(C) } 2023 \qquad \textbf{(D) } 2024 \qquad \textbf{(E) } 2025$

Problem 2

What is $10! - 7! \cdot 6!$?

$\textbf{(A) } -120 \qquad \textbf{(B) } 0 \qquad \textbf{(C) } 120 \qquad \textbf{(D) } 600 \qquad \textbf{(E) } 720$

Problem 3

For how many integer values of x is

$$|2x| \leq 7\pi?$$

$\textbf{(A) } 16 \qquad \textbf{(B) } 17 \qquad \textbf{(C) } 19 \qquad \textbf{(D) } 20 \qquad \textbf{(E) } 21$

Problem 4

Balls numbered $1, 2, 3, \ldots$ are placed in bins $A, B, C, D,$ and E so that the first ball is placed in $A,$ the next two are placed in $B,$ the next three are placed in $C,$ the next four are placed in D, the next five are placed in E, and then the next six go in A, etc. For example, $22, 23, \ldots, 28$ are placed in B. Which bin contains ball 2024?

$\textbf{(A)}\ A \qquad \textbf{(B)}\ B \qquad \textbf{(C)}\ C \qquad \textbf{(D)}\ D \qquad \textbf{(E)}\ E$

Problem 5

In the following expression, Melanie changed some of the plus signs to minus signs:

$$1+3+5+7+\cdots+97+99$$

When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 18$

Problem 6

A rectangle has integer side lengths and an area of 2024. What is the least possible perimeter of the rectangle?

$\textbf{(A)}\ 160 \qquad \textbf{(B)}\ 180 \qquad \textbf{(C)}\ 222 \qquad \textbf{(D)}\ 228 \qquad \textbf{(E)}\ 390$

Problem 7

What is the remainder when $7^{2024}+7^{2025}+7^{2026}$ is divided by 19?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 18$

Problem 8

Let N be the product of all the positive integer divisors of 42. What is the units digit of N?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Problem 9

Real numbers a, b and c have arithmetic mean 0. The arithmetic mean of a^2, b^2 and c^2 is 10. What is the arithmetic mean of ab, ac and bc?

$\textbf{(A)}\ -5 \qquad \textbf{(B)}\ -\frac{10}{3} \qquad \textbf{(C)}\ -\frac{10}{9} \qquad \textbf{(D)}\ 0 \qquad \textbf{(E)}\ \frac{10}{9}$

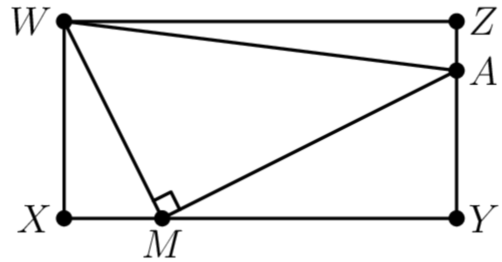
Problem 10

Quadrilateral $ABCD$ is a parallelogram, and E is the midpoint of the side $\overline{AD}$. Let F be the intersection of lines EB and AC. What is the ratio of the area of quadrilateral $CDEF$ to the area of triangle CFB?

$\textbf{(A)}\ 5:4 \qquad \textbf{(B)}\ 4:3 \qquad \textbf{(C)}\ 3:2 \qquad \textbf{(D)}\ 5:3 \qquad \textbf{(E)}\ 2:1$

Problem 11

In the figure below $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies $\overline{XY}$, point A lies on $\overline{YZ}$, and $\angle WMA$ is a right angle. The areas of $\triangle WXM$ and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?



(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Problem 12

A group of 100 students from different countries meet at a mathematics competition. Each student speaks the same number of languages, and, for every pair of students A and B, student A speaks some language that student B does not speak, and student B speaks some language that student A does not speak. What is the least possible total number of languages spoken by all the students?

(A) 9 **(B)** 10 **(C)** 12 **(D)** 51 **(E)** 100

Problem 13

Positive integers x and y satisfy the equation $\sqrt{x}+\sqrt{y}=\sqrt{1183}.$ What is the minimum possible value of $x+y$?

(A) 585 **(B)** 595 **(C)** 623 **(D)** 700 **(E)** 791

Problem 14

A dartboard is the region B in the coordinate plane consisting of points (x, y) such that $|x| + |y| \leq 8$. A target T is the region where $(x^2 + y^2 - 25)^2 \leq 49$. A dart is thrown at a random point in B. The probability that the dart lands in T can be expressed as $\frac{m}{n}\pi$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 39 **(B)** 71 **(C)** 73 **(D)** 75 **(E)** 135

Problem 15

A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, 7, as well as x, y, z with $x \leq y \leq z$. The range of the list is 7, and the mean and median are both positive integers. How many ordered triples (x, y, z) are possible?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** **infinitely many**

Problem 16

Jerry likes to play with numbers. One day, he wrote all the integers from 1 to 2024 on the whiteboard. Then he repeatedly chose four numbers on the whiteboard, erased them, and replaced them with either their sum or their product. (For example, Jerry's first step might have been to erase $1, 2, 3$, and 5, and then write either 11, their sum, or 30, their product, on the whiteboard.) After repeatedly performing this operation, Jerry noticed that all the remaining numbers on the board were odd. What is the maximum possible number of integers on the board at that time?

(A) 1010 **(B)** 1011 **(C)** 1012 **(D)** 1013 **(E)** 1014

Problem 17

In a race among 5 snails, there is at most one tie, but that tie can involve any number of snails. For example, the result of the race might be that Dazzler is first; Abby, Cyrus, and Elroy are tied for second, and Bruna is fifth. How many different results of the race are possible?

(A) 180 **(B)** 361 **(C)** 420 **(D)** 431 **(E)** 720

Problem 18

How many different remainders can result when the 100th power of an integer is divided by 125?

(A) 1 **(B)** 2 **(C)** 5 **(D)** 25 **(E)** 125

Problem 19

In the following table, each question mark is to be replaced by "Possible" or "Not Possible" to indicate whether a nonvertical line with the given slope can contain the given number of lattice points (points both of whose coordinates are integers). How many of the 12 entries will be "Possible"?

	zero	exactly one	exactly two	more than two
zero slope	?	?	?	?
nonzero rational slope	?	?	?	?
irrational slope	?	?	?	?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 9

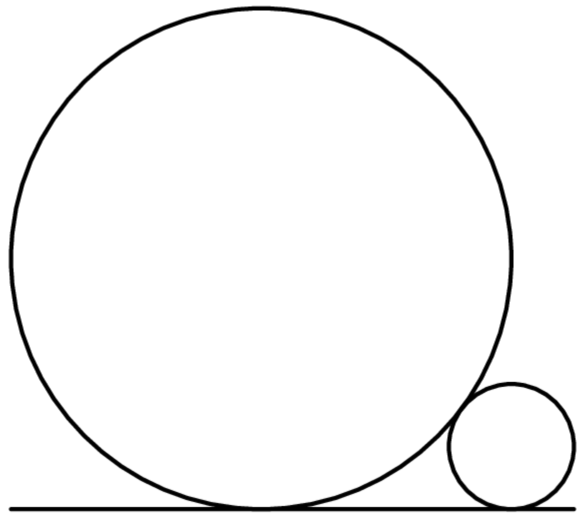
Problem 20

Three different pairs of shoes are placed in a row so that no left shoe is next to a right shoe from a different pair. In how many ways can these six shoes be lined up?

(A) 60 **(B)** 72 **(C)** 90 **(D)** 108 **(E)** 120

Problem 21

Two straight pipes (circular cylinders), with radii 1 and $\frac{1}{4}$, lie parallel and in contact on a flat floor. The figure below shows a head-on view. What is the sum of the possible radii of a third parallel pipe lying on the same floor and in contact with both?



(A) $\frac{1}{9}$ **(B)** 1 **(C)** $\frac{10}{9}$ **(D)** $\frac{11}{9}$ **(E)** $\frac{19}{9}$

Problem 22

A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M,$ where r and M are positive integers and M is not divisible by 3. What is r?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

Problem 23

The Fibonacci numbers are defined by $F_1 = 1,\ F_2 = 1,$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3.$ What Is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}}?$$

$\textbf{(A)}\ 318 \qquad \textbf{(B)}\ 319 \qquad \textbf{(C)}\ 320 \qquad \textbf{(D)}\ 321 \qquad \textbf{(E)}\ 322$

Problem 24

Let

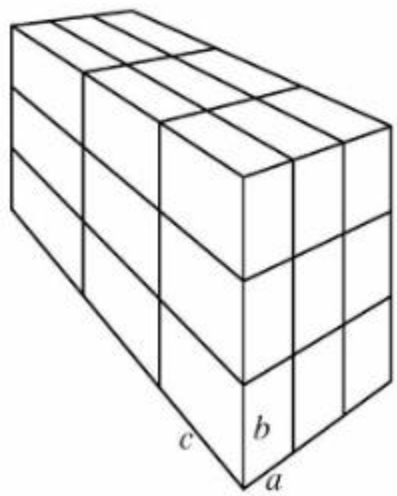
$$P(m) = \frac{m}{2} + \frac{m^2}{4} + \frac{m^4}{8} + \frac{m^8}{8}.$$

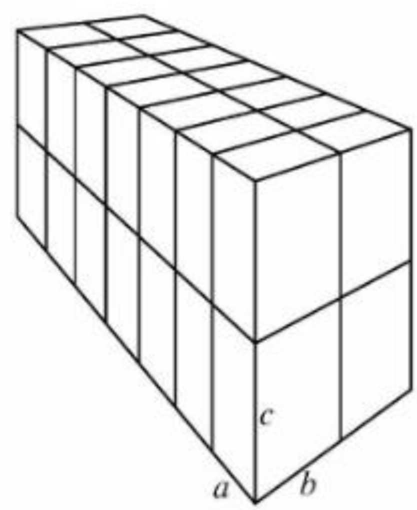
How many of the values of $P(2022)$, $P(2023)$, $P(2024)$, and $P(2025)$ are integers?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Problem 25

Each of 27 bricks (right rectangular prisms) has dimensions $a \times b \times c$, where a, b, and c are pairwise relatively prime positive integers. These bricks are arranged to form a $3 \times 3 \times 3$ block, as shown on the left below. A 28^{th} brick with the same dimensions is introduced, and these bricks are reconfigured into a $2 \times 2 \times 7$ block, shown on the right. The new block is 1 unit taller, 1 unit wider, and 1 unit deeper than the old one. What is $a + b + c$?





(A) 88 **(B)** 89 **(C)** 90 **(D)** 91 **(E)** 92