2019 AMC 12A Problems

Problem 1

The area of a pizza with radius 4 is N percent larger than the area of a pizza with radius 3 inches. What is the integer closest to N?

(A) 25 **(B)** 33 **(C)** 44 **(D)** 66 **(E)** 78

Problem 2

Suppose a is 150% of b. What percent of a is $3b$?

(A) 50 **(B)** $66\frac{2}{3}$ **(C)** 150 **(D)** 200 **(E)** 450

Problem 3

A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

(A) 75 **(B)** 76 **(C)** 79 **(D)** 84 **(E)** 91

Problem 4

What is the greatest number of consecutive integers whose sum is 45?

(A) 9 **(B)** 25 **(C)** 45 **(D)** 90 **(E)** 120

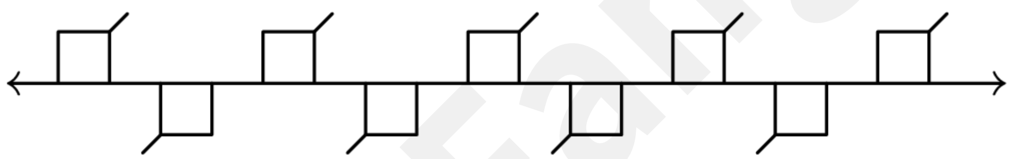
Problem 5

Two lines with slopes $\frac{1}{2}$ and 2 intersect at $(2,2)$. What is the area of the triangle enclosed by these two lines and the line $x+y=10$?

(A) 4 **(B)** $4\sqrt{2}$ **(C)** 6 **(D)** 8 **(E)** $6\sqrt{2}$

Problem 6

The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?

- some rotation around a point of line ℓ
- some translation in the direction parallel to line ℓ
- the reflection across line ℓ
- some reflection across a line perpendicular to line ℓ

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Problem 7

Melanie computes the mean μ, the median M, and the modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 1s, 12 2s, . . . , 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

(A) $\mu < d < M$ **(B)** $M < d < \mu$ **(C)** $d = M = \mu$ **(D)** $d < M < \mu$ **(E)** $d < \mu < M$

Problem 8

For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

(A) 14 **(B)** 16 **(C)** 18 **(D)** 19 **(E)** 21

Problem 9

A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and

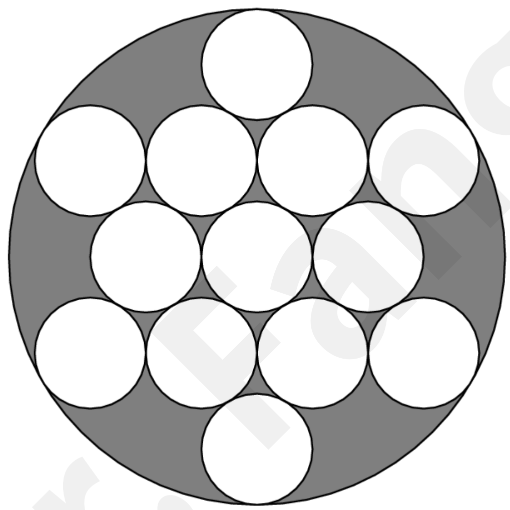
$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

for all $n \geq 3$ Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive inegers. What is $p + q$?

(A) 2020 **(B)** 4039 **(C)** 6057 **(D)** 6061 **(E)** 8078

Problem 10

The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?



(A) $4\pi\sqrt{3}$ **(B)** 7π **(C)** $\pi(3\sqrt{3}+2)$ **(D)** $10\pi(\sqrt{3}-1)$ **(E)** $\pi(\sqrt{3}+6)$

Problem 11

For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323..._k$. What is k?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Problem 12

Positive real numbers $x \neq 1$ and $y \neq 1$ satisfy $\log_2 x = \log_y 16$ and $xy = 64$. What is $(\log_2 \frac{x}{y})^2$?

(A) $\frac{25}{2}$ **(B)** 20 **(C)** $\frac{45}{2}$ **(D)** 25 **(E)** 32

Problem 13

How many ways are there to paint each of the integers $\mathbf{2, 3, \cdots, 9}$ either red, green, or blue so that each number has a different color from each of its proper divisors?

(A) 144 **(B)** 216 **(C)** 256 **(D)** 384 **(E)** 432

Problem 14

For a certain complex number c, the polynomial

$$P(x) = (x^2 - 2x + 2)(x^2 - cx + 4)(x^2 - 4x + 8)$$

has exactly 4 distinct roots. What is $|c|$?

(A) 2 **(B)** $\sqrt{6}$ **(C)** $2\sqrt{2}$ **(D)** 3 **(E)** $\sqrt{10}$

Problem 15

Positive real numbers a and b have the property that

$$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$$

and all four terms on the left are positive integers, where log denotes the base 10 logarithm. What is ab?

(A) 10^{52} **(B)** 10^{100} **(C)** 10^{144} **(D)** 10^{164} **(E)** 10^{200}

Problem 16

The numbers $1, 2, \ldots, 9$ are randomly placed into the 9 squares of a 3×3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

(A) $1/21$ **(B)** $1/14$ **(C)** $5/63$ **(D)** $2/21$ **(E)** $1/7$

Problem 17

Let s_k denote the sum of the kth powers of the roots of the polynomial $x^3 - 5x^2 + 8x - 13$. In particular, $s_0 = 3$, $s_1 = 5$, and $s_2 = 9$. Let a, b, and c be real numbers such that $s_{k+1} = a\,s_k + b\,s_{k-1} + c\,s_{k-2}$ for $k = 2, 3, \ldots$. What is $a + b + c$?

(A) -6 **(B)** 0 **(C)** 6 **(D)** 10 **(E)** 26

Problem 18

A sphere with center O has radius 6. A triangle with sides of length 15, 15, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

(A) $2\sqrt{3}$ **(B)** 4 **(C)** $3\sqrt{2}$ **(D)** $2\sqrt{5}$ **(E)** 5

Problem 19

In $\triangle ABC$ with integer side lengths,

$$\cos A = \frac{11}{16}, \qquad \cos B = \frac{7}{8}, \qquad \text{and} \qquad \cos C = -\frac{1}{4}.$$

What is the least possible perimeter for $\triangle ABC$?

(A) 9 **(B)** 12 **(C)** 23 **(D)** 27 **(E)** 44

Problem 20

Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval $[0,1]$. Two random numbers x and y are chosen independently in this manner. What is the probability that

$$|x-y| > \tfrac{1}{2}?$$

(A) $\frac{1}{3}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{2}{3}$

Problem 21

Let

$$z = \frac{1+i}{\sqrt{2}}.$$

What is

$$(z^{1^2} + z^{2^2} + z^{3^2} + \cdots + z^{12^2}) \cdot (\frac{1}{z^{1^2}} + \frac{1}{z^{2^2}} + \frac{1}{z^{3^2}} + \cdots + \frac{1}{z^{12^2}})?$$

(A) 18 **(B)** $72 - 36\sqrt{2}$ **(C)** 36 **(D)** 72 **(E)** $72 + 36\sqrt{2}$

Problem 22

Circles ω and γ, both centered at O, have radii 20 and 17, respectively. Equilateral triangle ABC, whose interior lies in the interior of ω but in the exterior of γ, has vertex A on ω, and the line containing side $\overline{BC}$ is tangent to γ. Segments $\overline{AO}$ and $\overline{BC}$ intersect at P, and $\dfrac{BP}{CP} = 3$. Then AB can be written in the form $\dfrac{m}{\sqrt{n}} - \dfrac{p}{\sqrt{q}}$ for positive integers m, n, p, q with $\gcd(m,n) = \gcd(p,q) = 1$. What is $m + n + p + q$?

(A) 42 **(B)** 86 **(C)** 92 **(D)** 114 **(E)** 130

Problem 23

Define binary operations $\diamondsuit$ and $\heartsuit$ by

$$a \diamondsuit b = a^{\log_7(b)} \qquad \text{and} \qquad a \heartsuit b = a^{\frac{1}{\log_7(b)}}$$

for all real numbers a and b for which these expressions are defined. The sequence (a_n) is defined recursively by $a_3 = 3 \heartsuit 2$ and

$$a_n = (n \heartsuit (n-1)) \diamondsuit a_{n-1}$$

for all integers $n \geq 4$. To the nearest integer, what is $\log_7(a_{2019})$?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

Problem 24

For how many integers n between 1 and 50, inclusive, is

$$\frac{(n^2-1)!}{(n!)^n}$$

an integer? (Recall that $0! = 1$.)

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

Problem 25

Let $\triangle A_0B_0C_0$ be a triangle whose angle measures are exactly 59.999°, 60°, and 60.001°. For each positive integer n define A_n to be the foot of the altitude from A_{n-1} to line $B_{n-1}C_{n-1}$. Likewise, define B_n to be the foot of the altitude from B_{n-1} to line $A_{n-1}C_{n-1}$, and C_n to be the foot of the altitude from C_{n-1} to line $A_{n-1}B_{n-1}$. What is the least positive integer n for which $\triangle A_nB_nC_n$ is obtuse?

(A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15