2024 AMC 12B Problems

Problem 1

In a long line of people, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

$\textbf{(A) } 2021 \qquad \textbf{(B) } 2022 \qquad \textbf{(C) } 2023 \qquad \textbf{(D) } 2024 \qquad \textbf{(E) } 2025$

Problem 2

What is $10! - 7! \cdot 6!$?

$\textbf{(A) } -120 \qquad \textbf{(B) } 0 \qquad \textbf{(C) } 120 \qquad \textbf{(D) } 600 \qquad \textbf{(E) } 720$

Problem 3

For how many integer values of x is

$$|2x| \leq 7\pi?$$

$\textbf{(A) } 16 \qquad \textbf{(B) } 17 \qquad \textbf{(C) } 19 \qquad \textbf{(D) } 20 \qquad \textbf{(E) } 21$

Problem 4

Balls numbered $1, 2, 3, \ldots$ are placed in bins $A, B, C, D,$ and E so that the first ball is placed in $A,$ the next two are placed in $B,$ the next three are placed in $C,$ the next four are placed in D, the next five are placed in E, and then the next six go in A, etc. For example, $22, 23, \ldots, 28$ are placed in B. Which bin contains ball 2024?

$\textbf{(A)}\ A \qquad \textbf{(B)}\ B \qquad \textbf{(C)}\ C \qquad \textbf{(D)}\ D \qquad \textbf{(E)}\ E$

Problem 5

In the following expression, Melanie changed some of the plus signs to minus signs:

$$1 + 3 + 5 + 7 + \cdots + 97 + 99$$

When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 15 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 27 \qquad \textbf{(E)}\ 28$

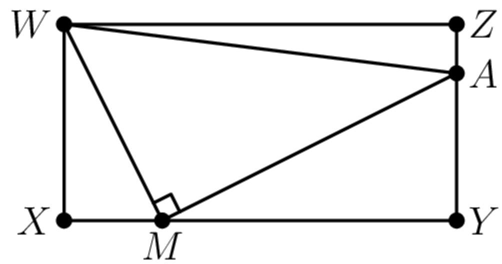
Problem 6

The national debt of the United States is on track to reach $5 \cdot 10^{1}3$ dollars by 2033. How many digits does this number of dollars have when written as a numeral in base 5? (The approximation of $\log_{10} 5$ as 0.7 is sufficient for this problem.)

$\textbf{(A)}\ 18 \qquad \textbf{(B)}\ 20 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 26$

Problem 7

In the figure below $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies $\overline{XY}$, point A lies on $\overline{YZ}$, and $\angle WMA$ is a right angle. The areas of $\triangle WXM$ and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?



$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 17$

Problem 8

What value of x satisfies

$$\frac{\log_2 x \cdot \log_3 x}{\log_2 x + \log_3 x} = 2?$$

$\textbf{(A)}\ 25 \qquad \textbf{(B)}\ 32 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 42 \qquad \textbf{(E)}\ 48$

Problem 9

A dartboard is the region B in the coordinate plane consisting of points (x, y) such that

$$|x| + |y| \leq 8.$$

A target T is the region where

$$(x^2 + y^2 - 25)^2 \leq 49.$$

A dart is thrown at a random point in B. The probability that the dart lands in T can be expressed as $\frac{m}{n}\pi$, where m and n are relatively prime positive integers. What is $m+n$?

$\textbf{(A)}\ 39 \qquad \textbf{(B)}\ 71 \qquad \textbf{(C)}\ 73 \qquad \textbf{(D)}\ 75 \qquad \textbf{(E)}\ 135$

Problem 10

A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, 7, as well as x, y, z with $x \le y \le z$. The range of the list is 7, and the mean and median are both positive integers. How many ordered triples (x, y, z) are possible?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ \textbf{infinitely many}$

Problem 11

Let $x_n = \sin^2(n^\circ)$. What is the mean of $x_1, x_2, x_3, \cdots, x_{90}$?

$\textbf{(A)}\ \frac{11}{45} \qquad \textbf{(B)}\ \frac{22}{45} \qquad \textbf{(C)}\ \frac{89}{180} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{91}{180}$

Problem 12

Let z be a complex number with real part greater than 1 and $|z| = 2$. In the complex plane, the four points 0, z, z^2, and z^3 are the vertices of a quadrilateral with area 15. What is the imaginary part of z?

$\textbf{(A)}\ \frac{3}{4} \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ \frac{4}{3} \qquad \textbf{(D)}\ \frac{3}{2} \qquad \textbf{(E)}\ \frac{5}{3}$

Problem 13

There are real numbers x, y, h and k that satisfy the system of equations

$$x^2 + y^2 - 6x - 8y = h$$

$$x^2 + y^2 - 10x + 4y = k$$

What is the minimum possible value of $h + k$?

$\textbf{(A)}\ -54 \qquad \textbf{(B)}\ -46 \qquad \textbf{(C)}\ -34 \qquad \textbf{(D)}\ -16 \qquad \textbf{(E)}\ 16$

Problem 14

How many different remainders can result when the 100th power of an integer is divided by 125?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 25 \qquad \textbf{(E)}\ 125$

Problem 15

A triangle in the coordinate plane has vertices $A(\log_2 1, \log_2 2)$, $B(\log_2 3, \log_2 4)$, and $C(\log_2 7, \log_2 8)$. What is the area of $\triangle ABC$?

$\textbf{(A)}\ \log_2 \frac{\sqrt{3}}{7} \qquad \textbf{(B)}\ \log_2 \frac{3}{\sqrt{7}} \qquad \textbf{(C)}\ \log_2 \frac{7}{\sqrt{3}} \qquad \textbf{(D)}\ \log_2 \frac{11}{\sqrt{7}} \qquad \textbf{(E)}\ \log_2 \frac{11}{\sqrt{3}}$

Problem 16

A group of 16 people will be partitioned into 4 indistinguishable 4 -person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 7 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 9$

Problem 17

Integers a and b are randomly chosen without replacement from the set of integers with absolute value not exceeding 10 . What is the probability that the polynomial $x^3 + ax^2 + bx + 6$ has 3 distinct integer roots?

$\textbf{(A)}\frac{1}{240} \qquad \textbf{(B)}\frac{1}{221} \qquad \textbf{(C)}\frac{1}{105} \qquad \textbf{(D)}\frac{1}{84} \qquad \textbf{(E)}\frac{1}{63}$.

Problem 18

The Fibonacci numbers are defined by $F_1 = 1,\ F_2 = 1,$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3.$ What Is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}}?$$

$\textbf{(A)}\ 318 \qquad \textbf{(B)}\ 319 \qquad \textbf{(C)}\ 320 \qquad \textbf{(D)}\ 321 \qquad \textbf{(E)}\ 322$

Problem 19

Equilateral triangle ABC with side length 14 is rotated about its center by angle θ to form $\triangle DEF$, where $0 < \theta < 60^\circ$. The area of hexagon $ADBECF$ is $91\sqrt{3}$. What is $\tan\theta$?

$\textbf{(A)}\ \frac{3}{4} \qquad \textbf{(B)}\ \frac{5\sqrt{3}}{11} \qquad \textbf{(C)}\ \frac{4}{5} \qquad \textbf{(D)}\ \frac{11}{13} \qquad \textbf{(E)}\ \frac{7\sqrt{3}}{13}$

Problem 20

Suppose A, B, and C are points in the plane with $AB = 40$ and $AC = 42$, and let x be the length of the line segment from A to the midpoint of $\overline{BC}$. Define a function f by letting $f(x)$ be the area of $\triangle ABC$. Then the domain of f is an open interval (p, q), and the maximum value r of $f(x)$ occurs at $x = s$. What is $p + q + r + s$?

$\textbf{(A)}\ 909 \qquad \textbf{(B)}\ 910 \qquad \textbf{(C)}\ 911 \qquad \textbf{(D)}\ 912 \qquad \textbf{(E)}\ 913$

Problem 21

The measures of the smallest angles of three different right triangles sum to 90°. All three triangles have side lengths that are primitive Pythagorean triples. Two of them are $3-4-5$ and $5-12-13$. What is the perimeter of the third triangle?

$\textbf{(A)}\ 40 \qquad \textbf{(B)}\ 126 \qquad \textbf{(C)}\ 154 \qquad \textbf{(D)}\ 176 \qquad \textbf{(E)}\ 208$

Problem 22

Let $\triangle ABC$ be a triangle with integer side lengths and the property that $\angle B = 2\angle A$. What is the least possible perimeter of such a triangle?

$\textbf{(A)}\ 13 \qquad \textbf{(B)}\ 14 \qquad \textbf{(C)}\ 15 \qquad \textbf{(D)}\ 16 \qquad \textbf{(E)}\ 17$

Problem 23

A right pyramid has regular octagon $ABCDEFGH$ with side length 1 as its base and apex V. Segments AV and DV are perpendicular. What is the square of the height of the pyramid?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ \frac{1+\sqrt{2}}{2} \qquad \textbf{(C)}\ \sqrt{2} \qquad \textbf{(D)}\ \frac{3}{2} \qquad \textbf{(E)}\ \frac{2+\sqrt{2}}{3}$

Problem 24

What is the number of ordered triples (a,b,c) of positive integers, with $a \le b \le c \le 9$, such that there exists a (non-degenerate) triangle $\triangle ABC$ with an integer inradius for which a , b , and c are the lengths of the altitudes from A to $\overline{BC}$, B to $\overline{AC}$, and C to $\overline{AB}$, respectively? (Recall that the inradius of a triangle is the radius of the largest possible circle that can be inscribed in the triangle.)

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

Problem 25

Pablo will decorate each of 6 identical white balls with either a striped or a dotted pattern, using either red or blue paint. He will decide on the color and pattern for each ball by flipping a fair coin for each of the 12 decisions he must make. After the paint dries, he will place the 6 balls in an urn. Frida will randomly select one ball from the urn and note its color and pattern. The events "the ball Frida selects is red" and "the ball Frida selects is striped" may or may not be independent, depending on the outcome of Pablo's coin flips. The probability that these two events are independent can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is m? (Recall that two events A and B are independent if $P(A \text{ and } B) = P(A) \cdot P(B)$.)

$\textbf{(A)}\ 243 \qquad \textbf{(B)}\ 245 \qquad \textbf{(C)}\ 247 \qquad \textbf{(D)}\ 249 \qquad \textbf{(E)}\ 251$

2024 AMC 12B Answer Key

1. B
2. B
3. E
4. D
5. B
6. B
7. C
8. C
9. B
10. C
11. E
12. D
13. C
14. B
15. B
16. A
17. C
18. B
19. B
20. C
21. C
22. C
23. B
24. B
25. A